Homework #2: Due April 18st

1) View the video at https://www.youtube.com/watch?v=zkr3JmhjKbg then answer the following questions:
   1. Why is the electron source normally at the top?
   2. In the video the sample holder is touched with hands -- is this a good idea?
   3. What is the lowest aperture used for?
   4. Before he adjusts the focus knob (~11:30 in the video), is the sample underfocused or overfocused?

2) View the video at https://www.youtube.com/watch?v=_q7wKmV9-7c then answer the following questions:
   1. Why do SEM images often contain shadows, similar to light images?
   2. How does the microscope voltage change the depth resolution?

The videos at https://www.youtube.com/watch?v=spUNpyF58BY and https://www.youtube.com/watch?v=xhO8iz2qCOE , other similar or your math notes from other classes may be useful revision. (Note: he does not use the same notation as I do.)

3) Consider the integral $\frac{1}{R}\int_0^R \exp(2\pi iux)dx$ with $u \neq 0$ and $R = n/u$ with n an integer. Split the integration into the ranges $0 \to \frac{1}{u}, \frac{1}{u} \to \frac{2}{u}, ... \frac{n-1}{u} \to R$ and work out the result of each. By summing the ranges show that the result is zero. We use this quite a lot.

4) Consider as a definition $\int f(x)\exp(2\pi iux)dx = F(u)$. (For reference, the "2π" is standard for Fourier Transforms in diffraction.) By substituting y=x-a show that (being careful about how you substitute inside an integral

   $$\int f(x-a)\exp(2\pi iux)dx = \exp(2\pi iua)F(u)$$

   This shows that a shift of the origin (here along x) leads to a exponential multiplier. Again, we will use this result.

5) *(Harder)* With the definitions $\int f(x)\exp(2\pi iux)dx = F(u)$ and $\int g(x)\exp(2\pi iux)dx = G(u)$,

   consider $\int F(u)G(u)\exp(-2\pi iux)du$. By writing this as

$$\int F(u)G(u)exp(-2\pi iux)du = \int \left\{\int f(y)exp(2\pi iuy)dy\right\}\left\{\int g(z)exp(2\pi iuz)dz\right\} exp(-2\pi iux)du$$

separate out the integration over u and show that $\int F(u)G(u)\exp(-2\pi iux)du = \int f(x-y)g(y)dy$

This is called a convolution, and has an important role in simplifying many elements of diffraction.