Short note

Is there an electron wind?

L D Marks and J P Zhang

Center for Surface Radiation Damage Studies, Department of Materials Science and Engineering, Northwestern University, Evanston, IL 60208, USA

Received 26 December 1991

The question of whether there is any momentum transfer to a specimen during electron diffraction, as dictated by Newton's third law, is discussed. It is pointed out that there must be momentum and energy transfer during "elastic" scattering. Even though what is normally considered as "elastic" scattering is in fact inelastic, electron density systems are insensitive to the changes in energy. Therefore the energy changes during diffraction do not prevent coherent interference phenomena such as electron holography and high-resolution electron microscopy. The momentum transfer, in effect, introduces an electron wind which can, mechanically, alter the specimen. The wind is equivalent to a storm with gusts of up to 300 km/h. Experimental evidence in support of this is presented.

1. Introduction

A not uncommon question in electron microscopy is what happens to the momentum transferred by the electron beam to a crystal. If the beam passes through a crystal and is preferentially diffracted in one direction, is the momentum "lost" by the beam transferred to the crystal? Newton's third law dictates that this must be the case. Some experimental observations provide circumstantial evidence for this, for instance, with small particles, if the particles are supported on the top surface of a film they often do not line up on the zone axis, but if they are on the bottom they do. As illustrated in Fig. 1, this can be understood in terms of a momentum transfer [1].

As a second example, during quasi-melting of small particles [2-4] many good zone-axis images are observed (which statistically is surprising), and momentum transfer could be assisting either the orientation or the particle restructuring [5]. However, if momentum is transferred to the crystal, then we are dealing with inelastic scattering, not elastic scattering. In addition, inelastic scattering is normally considered as incoherent, and the inelastic wave does not coherently interfere with the elastic component. However, electron holography and high-resolution electron microscopy work, so the wave passing through a specimen must be coherent with the wave that does not pass through the specimen.

Mathematically, this issue is often avoided by considering an infinitely large crystal. The mo-
momentum is transferred to the crystal as a whole, and energy difference is (in the limit) zero However, infinite limits always introduce problems, and even if the energy transfer in the limit vanishes, it does not follow that the momentum transfer, which will lead to a pressure, also vanishes. Furthermore, electron microscope specimens are never infinite.

In this note we consider a little more carefully this question. It is pointed out that the momentum transfer leads to an energy change with a definite phase relationship to the true elastic component. Although, in principle, this makes the transmitted and diffracted beams incoherent, electron detectors operate within a sufficiently short time scale that the energy change is undetectable. Experimental evidence for a significant role for the momentum transfer in the orientation of \( \text{BaO} \) produced by electron-beam-induced decomposition of \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) is presented.

2. Analysis

We will start by establishing that the energy loss during diffraction is not detectable with standard electron detectors, and can therefore be neglected. Let us consider the very simple problem of an electron being scattered from a single atom, ignoring relativistic effects. In the lab frame, we take the electron wavevector as \( \mathbf{k} \) with the atom initially at rest with a wavevector of \( \mathbf{K} \). We can move to the center of mass frame by subtracting \( \frac{\mathbf{k} M}{m_e + M} \) from each, which gives the reduced electron wavevector

\[
k' = \frac{\mathbf{k} m}{m + M},
\]

(1)

and the reduced atom wavevector

\[
K' = -\frac{\mathbf{k} M}{m + M}
\]

(2)

Introducing a scattering event which transfers a wavevector \( s \),

\[
k' \rightarrow k' + s,
\]

(3)

\[
K' \rightarrow K' - s
\]

(4)

Going back to the lab frame, for the electron we have

\[
k'' = k + s,
\]

(5)

and for the atom

\[
K'' = -s,
\]

(6)

which corresponds to a change in momentum of \( \hbar s \). The energy of the atom is therefore \( \hbar^2 s^2 / 2M \), and that of the electron \( E_0 - \hbar^2 s^2 / 2M \).

The outgoing electron wave, including a range of scattering vectors \( s \), can be written as

\[
\psi(r, t) = \int P(s) \exp(2\pi i [k + s] \cdot r) + i\hbar t s^2 / 2M) \, ds,
\]

(7)

where we have incorporated the energy transfer. The critical point is that the electron wave is detected by a process which is inherently an energy loss process, for instance by exciting a phosphor to produce electron/hole pairs which later recombine to produce photons. This process takes a short time, typically of the order of \( 10^{-15} \) s or less. The intensity integrated over this time scale (ignoring terms from the microscope lenses for simplicity) will be

\[
I(r) = 10^{15} \int \int P(s) P^*(s') \times \exp(2\pi i [s - s'] \cdot r) + i\hbar t [s^2 - s'^2] / 2M) \, ds \, ds' \, dt,
\]

(8)

where we have multiplied by \( 10^{15} \) to take account of the time range \( t \). As an estimate, taking \( s = 0 \) and \( s' = 10 \text{ nm}^{-1} \), we can separate out the time integral and write

\[
I(r) = \int P(s) P^*(s') \exp(2\pi i [s - s'] \cdot r) \, ds \, ds' \times 10^{15} \int_{0}^{10^{-15}} \exp(2\pi i t \delta \times 10^{12} / Z) \, dt,
\]

(9)

where \( Z \) is the atomic mass (in amu). The time integral is effectively unity, and does not enter
L D Marks, J P Zhang / Is there an electron wind?

Fig 2 High-resolution images of the 123 superconductor for a number of different orientations. In all cases the BaO at the surface on the amorphous intermediary is aligned with either a [100] or [110] zone axis parallel to the incident beam. In (a), a threefold zone is shown, in (b) there is no systematic zone apparent in the substrate.

Fig 3 High-resolution image of ReO₃ showing numerous small fcc particles on the surface due to reaction with carbon. More details are given in ref [7].

Table 1

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>(110)</th>
<th>(100)</th>
<th>(uvw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₃</td>
<td>19</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>C₂</td>
<td>21</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>None</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>67</td>
<td>26</td>
<td>14</td>
</tr>
<tr>
<td>%</td>
<td>63</td>
<td>24</td>
<td>13</td>
</tr>
</tbody>
</table>

(uvw) indicates higher-index zones and the rotational symmetry of the substrate is indicated on the left.

into the intensity analysis. Waves with different scattering vectors are incoherent, but the detection time is so short that the incoherence is undetected. Therefore, it is valid to ignore the energy (momentum) transfer to the crystal when considering the final image intensity. The above arguments can be extended to a crystal simply by changing the effective mass, and the larger the crystal the stronger this argument will be.

3. Experimental evidence

Complete experimental demonstration of the electron wind would require mechanical measurements, for instance the change in bend contour spacing as a function of electron flux, but uniqueness would be difficult since beam heating effects are possible. We will present here evidence which we believe can only be explained by an electron wind. During electron beam irradiation of the 123 superconductors [6] a surface amorphous phase forms, followed by BaO at the surface, presumably oxygen is being lost in the beam and the material is phase separating. If the electron beam has a mechanical effect, one would expect to find the BaO preferentially oriented along a low-index zone axis, independent of the initial orientation of the superconductor. This is the experimental result as shown in fig 2. To verify that this is not an artifact, it is important to consider the statistical probability of seeing a...
zone-axis image of a small particle. Assuming that a zone-axis image can be seen if the tilt is within 50 mrad of the zone-axis orientation, the probability of seeing a [111]-oriented particle is ~ 6% and a [001]-oriented particle ~ 5%, these numbers are significantly smaller than the experimental results (see table 1). A second, more visual, example shown in fig 3 is from the carbon-induced reduction of ReO$_3$ [7], numerous small fcc particles (probably of ReC) are formed almost all of which are [110] oriented.

4. Discussion

The apparent problem with including an energy transfer component to elastic diffraction to take account of the momentum transfer is not, in fact, a problem. Elastic diffraction is in reality inelastic, but the energy change is so small that at the detector it is negligible. In principle one could design an experiment to detect the energy change, for instance using a light material for the scattering and a very slow detector.

An interesting question is how substantial is the force generated by diffraction. In this context it is important to note that although the energy transfer may be vanishingly small, due to the large mass of the crystal, the momentum transfer does not depend upon the mass. As an order-of-magnitude estimate, consider a 10 nm cube, an incident beam flux of $10^4$ electrons per ångström squared per second (a typical value for high-resolution electron microscopy) and that the electrons are all diffracted by a scattering (diffraction) vector of 10 nm$^{-1}$. The mean force, $F$, on the particle is

$$F = hs \times \text{Flux} \times \text{Area} = 6.6 \times 10^{-17} \text{N}$$

Taking this force over one face is equivalent to a pressure of 0.66 N/m$^2$ which is fairly modest (This pressure is approximately equivalent to a wind (in air) of velocity 2 miles/h or 3 km/h.) However, this is only the mean pressure, and the electrons are in fact well separated in time so that the peak pressure and force will be substantially larger, and this may easily affect a specimen. Physically, it is not unreasonable to consider the electron beam as equivalent to gusts of wind with velocities of up to 300 km/h. It does, therefore, seem to be quite reasonable to think of an electron "wind" which can have noticeable effects on microscope specimens.

5. Conclusions

Newton's third law is conserved in electron diffraction, when the electron momentum changes the crystal also changes momentum. Plane wave components of the electron beam with different scattering vectors are, strictly speaking, incoherent with respect to each other. However, electron detectors are insensitive to this. Similar to light pressure, there is an electron pressure which is not negligible.

Acknowledgements

This work was supported by the Science and Technology Center for Superconductivity on grant no NSF/DMR-8809854. The authors would like to thank Ms. R Ai for assistance with the work on ReO$_3$.

References

[1] L.D Marks and A Howie, 1977, unpublished. Similar calculations were also performed by M.J. Treacy and M Brown at about the same time, also unpublished.