Electron Crystallography in Surface Structure Analysis

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ABSTRACT Surface structure analysis is an important area of research, and in recent years notable advances have been made in this field, both in improved techniques for studying surfaces and in methods of analyzing them. This review aims to summarize the techniques available, particularly those relating to electron microscopy, and also to outline one of the newest areas of development, the application of direct methods to surface structure analysis. Microsc. Res. Tech. 46:160–177, 1999. © 1999 Wiley-Liss, Inc.

INTRODUCTION

It has been known for many years that the chemical and physical properties of a material are dependent on the crystal structure of the compound (Barker and Auerbach, 1984; Hudson, 1992; Zangwill, 1988). The development of direct methods of phasing for X-ray (Hauptman and Karle, 1953; Woolfson and Fan, 1995) and electron diffraction (Dorset, 1995; Vainshtein, 1964) analysis have made crystallography a flourishing field in science. The use of direct methods in surface structure analysis has, however, only begun to develop recently (Landree et al., 1997; Marks and Landree, 1997; Marks et al., 1997). This review aims to summarize the progress made in this field.

A surface acts as an interface between two phases, and this interface may exhibit properties which control the behavior of the system. These properties are a direct result of the surface structure, and, in order to design systems that utilize these fully, the surface structure should be known (Somorjai and Starke, 1992; Venables et al., 1984). A surface structure is not, however, easy to predict. While it may share some characteristics with that of the bulk, the atomic positions are much more influenced by the external environment. If the bulk crystal structure arrangement simply stops at the surface, the atoms are left with fewer neighbors, and therefore a high-energy, low-stability system is formed. In general, this leads to some form of reorganization, where the atoms rearrange themselves to positions of lower energy. Such realignment of atoms to energetically favorable positions can extend for several layers into the bulk, and this too must be correctly identified if the properties of the material are to be fully understood.

Despite rearranging to the lowest energy state possible, the atoms on a surface are still of relatively high energy, and thus more likely to react with other species. The rate at which a monolayer of a species covers a surface is pressure-dependent (Somorjai, 1995); for a material with a sticking coefficient of unity, a monolayer will be deposited in approximately 3 seconds at a pressure of $10^{-6}$ torr. If the pressure is reduced to $10^{-9}$ torr however, the deposition time rises to around an hour (Prutton, 1994). Thus, in order to study “clean” surfaces, Ultra High Vacuum (UHV) systems, capable of $10^{-10}$ torr or better, must be used. These were developed in the 1960s, and the development of techniques to study surfaces has run in parallel with improvement of these systems. In recent years UHV systems that have transmission electron microscope facilities attached have been developed, and this has opened up a new area of surface investigation (Bonevich and Marks, 1992; Collazo-Davila et al., 1995; Jayaram et al., 1995; Takayanagi et al., 1978).

The quality of the sample is of the utmost importance in such studies. In general, the specimen is first treated by some combination of dimpling and mechanical tapping or polishing to create regions that will be suitable for electron beam studies. Once this is achieved, the sample is placed in a UHV system, where it undergoes further cleaning, usually by a combination of ion beam bombardment and annealing, until a reproducible clean surface is obtained. This method of preparation generally works well, although in some cases the ions induce specimen damage, which is difficult to overcome (Dunn et al., 1991; Marks et al., 1991). It is important to monitor the sample surface during the experimental procedure, since even at low pressures residual gases are present that may react with the surface. Monitoring is most commonly achieved using Auger electron spectroscopy (AES) to track the chemical composition of the surface (Weissman and Müller, 1981).

Many techniques now exist for studying surfaces, although quantitative analysis of the actual surface structure is still a very specialized field. Scanning tunneling microscopy (STM) can show where areas of electron density lie at the surface (see, for example, van de Leemput and van Kempen, 1992), but identifying atomic type is difficult (Spence et al., 1996), and it does...
Fig. 1. Comparison of various surface structure analysis experimental techniques. a: Low energy electron diffraction (LEED). b: Reflection high energy electron diffraction (RHEED). c: Photoelectron holography (PEH). d: Transmission electron diffraction (TED).
not assist in the identification of subsurface rearrangements. Techniques such as AES (Turner and Schreffels, 1994; Mroz, 1996), X-ray photoelectron spectroscopy (XPS) (Grimblot et al., 1990; Turner and Schreffels, 1994), secondary ion mass spectrometry (SIMS) (Greenlieff and White, 1993), and Rutherford backscattering (RBS) (Chu and Liu, 1996) give information about the chemical composition and chemical configuration of the atoms at the surface. These techniques do not give any detail as to the atomic positions, nor do they provide any information about the layers below the surface, which could prove important. To obtain information about atomic positions on the surface and the surface substructure, diffraction techniques must be utilized. The results from analysis of diffraction data from a surface may be used directly or combined with the results from the techniques already mentioned to solve the surface structure. Both X-ray (Feidenhansl, 1989; Robinson and Tweet, 1992) and electron diffraction (J ayaram et al., 1993, 1995; Takayanagi et al., 1995b) techniques can be used, but for the purposes of this review only experimental electron diffraction techniques will be mentioned in any detail.

**EXPERIMENTAL METHODS**

**Low Energy Electron Diffraction (LEED)**

LEED analysis has been widely used as a tool in surface structure analysis for many years (Heinz, 1988, 1995; J ona et al., 1982; Van Hove, 1997). Electrons of approximately 20–500 eV energy are easily scattered by atoms, but if they penetrate into a solid by more than approximately six atomic layers, they are adsorbed. If, however, they are back-scattered out of the sample, these electrons will provide information about the atomic positions, symmetry, and, in many cases, periodicity of the structure concerned.

The experimental procedure is relatively straightforward (see Fig. 1a). An electron gun produces a monochromatic electron beam, incident on the specimen, and a detector measures the diffracted beams. Both sharp spot (LEED) and diffuse (DLEED) electron-diffraction patterns may be obtained, where the type of pattern is dependent on the surface order present. Structure analysis is possible from both types of patterns. In order to obtain information about the atomic positions, the intensities of the diffracted beams must be considered. This is not straightforward, since simple kinematic scattering theory is not sufficient, and dynamical models are required. For LEED, dynamical scattering theory (Pendry, 1974, 1994; Tong, 1994) is used to obtain plots of the intensity of a beam against its energy on an I-V curve, and structural analysis is then achieved by comparing the experimental results to these plots. DLEED analysis has developed relatively recently (Heinz, 1994, 1995; Pendry and Saldin, 1984), and follows the same procedure as that for LEED. An advantage of DLEED is that it can also be used to produce two-dimensional intensity maps, which may be used as electron holograms (Saldin and De Andres, 1990) to reproduce the surface structure directly. This does not work for single energy analysis (Wei and Tong, 1992), however, and large data sets are required in order to produce comprehensible results, limiting the usefulness of this technique.

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**Table 1. Surface Structures That Have Been Studied Using the MRE Method**

<table>
<thead>
<tr>
<th>Structure</th>
<th>Data</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si(111)-(3×3)</td>
<td>R30°Au</td>
<td>Electron Marks et al. (1997)</td>
</tr>
<tr>
<td>Si(111)-(5×2)</td>
<td>Au</td>
<td>Electron Marks et al. (1997)</td>
</tr>
<tr>
<td>Si(111)-(7×7)</td>
<td></td>
<td>Electron Gilmore et al. (1997)</td>
</tr>
<tr>
<td>Si(111)-(4×1)</td>
<td>In</td>
<td>Electron Colazzo-Davila et al. (1997)</td>
</tr>
<tr>
<td>Si(111)-(6×6)</td>
<td>Au</td>
<td>X-ray Marks et al. (1998a)</td>
</tr>
<tr>
<td>TiO&lt;sub&gt;2&lt;/sub&gt;(100)×3</td>
<td></td>
<td>X-ray Landree et al. (1998)</td>
</tr>
<tr>
<td>Si(111)-(3×1)</td>
<td>Ag</td>
<td>Electron Grozea et al. (1999)</td>
</tr>
<tr>
<td>Si(111)-(3×3)</td>
<td>R30°Ag</td>
<td>Electron Colazzo-Davila et al. (1998)</td>
</tr>
<tr>
<td>Ge(111)-(4×4)</td>
<td>Ag</td>
<td>X-ray (1998b)</td>
</tr>
<tr>
<td>MgO(111)-(3×3)</td>
<td>R30°</td>
<td>Electron Plass et al. (1998)</td>
</tr>
<tr>
<td>MgO(111)-(2×2)</td>
<td></td>
<td>Electron Plass et al. (1998)</td>
</tr>
<tr>
<td>MgO(111)-(2×2, 3) R30°</td>
<td></td>
<td>Electron Plass et al. (1998)</td>
</tr>
<tr>
<td>Ni(111)-(5,2×2)</td>
<td>S</td>
<td>X-ray Marks et al. (1998b)</td>
</tr>
<tr>
<td>Cu(111)-4×1</td>
<td>1×1</td>
<td>X-ray Marks et al. (1998b)</td>
</tr>
<tr>
<td>Cu(110)-p(4×1)</td>
<td>8i</td>
<td>X-ray Marks et al. (1998b)</td>
</tr>
</tbody>
</table>

The main drawback to LEED analysis is that it is a “trial and error” method, in which models are proposed and compared to the experimental data until a fit between the two is found. Advances have been made in recent years in the reduction of time involved in calculating intensities (tensor LEED) (Rous, 1992; Rous and Pendry, 1989a,b; Rous et al., 1986), and the development of directed search methods to improve the starting models (linear LEED) (Rous et al., 1990; Wander et al., 1992). While this has improved the timescale of an analysis, the fact remains that a poor initial model structure will make a LEED analysis particularly complex.

**Reflection High Energy Electron Diffraction (RHEED)**

In the RHEED technique, a high energy electron beam of energy approximately 3–100 keV is angled towards the specimen surface. The electrons are diffracted by the sample surface, and picked up by a recording device positioned opposite the source (see Fig. 1b). The diffraction pattern consists of a series of streaks, with the sharpness of each streak related to the roughness of the surface on the atomic scale (Ichimiya et al., 1997; Korte and Meyerehmsen, 1993; Ma and Marks, 1990; Maksym, 1985). As with LEED, dynamical scattering theory must be applied in order to understand the diffraction pattern fully (Ichimiya et al., 1993; Ma and Marks, 1992; Maksym and Beeby, 1981; Peng and Cowley, 1986; Peng et al., 1996). Plotting the intensity of the diffracted beams against the angle of incidence gives a rocking curve, and these curves may be used to analyze atomic positions, since the intensity of a diffracted beam is a function of the three-dimensional structure (Nakahara and Ichimiya, 1991; Hashizume et al., 1994). Azimuthal plots and analysis of the intensity distribution in a RHEED pattern are also used in structure determination (Mitura and Maksym, 1993; Hashizume et al., 1995).

The technique is not without problems (Price, 1992). The diffraction pattern obtained is a projection of the reciprocal lattice, and so the sample must be rotated to give information about a second projection. This can prove difficult. Interpretation of the data is not straight-
forward, and information can be lost from the diffraction pattern due to incorrect analysis of the streaks. Thus, while RHEED is capable of structural analysis (Ichimiya et al., 1997), it is a fairly complicated and time-consuming method, which in practice is used more as a monitor of surface roughness, for example in molecular beam epitaxy (MBE) studies (Price, 1992), or to monitor the level of contamination on a surface, though AES is more commonly used for this purpose.
Photoelectron Holography (PEH)

A photoelectron hologram is a two-angle photoelectron diffraction pattern that can be reconstructed to create a three-dimensional image of the structure of a solid surface from which the diffraction data were obtained (Szőke, 1986). The source of the coherent electrons is a group of atoms located inside the target object (see Fig. 1c). An atom absorbs a photon and produces a photoelectron as the reference wave. The object is assumed to scatter these coherent electrons only weakly, and thus the incident reference-wave amplitude is largely unaffected by passing through the object. The limited interaction between the reference wave and the object produces scattered waves, however, and these interfere with the reference wave to produce the diffraction pattern or hologram. This may then be Fourier transformed to produce an atomic image of the local surface area (Barton, 1990; Fadley et al., 1997; Len et al., 1995; Reuter et al., 1997; Saldin, 1997).

PEH is a relatively new technique, and as such it is difficult to comment on its effectiveness. So far for the systems it has been used to study, it has proved to be an extremely useful tool, although these systems have not, as yet, been particularly complex. In terms of experimental requirements, PEH is perhaps one of the more user-friendly surface analysis methods, and it may prove to be a popular and reliable technique for determining surface structures in the future.

Transmission Electron Microscopy (TEM) and Diffraction (TED)

The previously discussed techniques are limited to studying the first few layers of a specimen. While this is advantageous in many ways, it does mean that bulk interactions with the surface are not accessible. TEM and TED studies, like X-ray diffraction analysis, allow bulk and surface structures to be considered at the same time (see Fig. 1d).

Images obtained from TEM studies provide useful structural information (Cowley, 1986; Gibson et al., 1997; Williams and Marks, 1995). Profile imaging (Ikarashi et al., 1988; Marks, 1983, 1984; Marks and Smith, 1983), in which the incident beam is parallel to the surface of interest, produces images that show...
atomic positions on the surface, but it is difficult to obtain reproducible results, due to the thermodynamic instability of a thin edge. Plan view imaging, in which the incident beam is perpendicular to the surface, can be used either on or off a zone axis, and can provide images that give important phase information, as well as atomic-scale structural information (Dunn et al., 1991; Marks, 1992b; Marks et al., 1992, 1993; Nihoul et al., 1984).

Collection of TED data is a relatively straightforward process, and is usually carried out via a series of exposures on photographic film, CCD or image plate recording. TED patterns yield the intensities of the diffraction spots, as well as partial information about the unit cell symmetry and dimensions (Jayaram et al., 1993; Marks, 1992a; Xu and Marks, 1992a,b), but the intensity data must be reduced to overcome both the problems of diffuse scattering in a diffraction pattern and the low signal-noise ratio of surface diffraction spots. This is done by means of cross-correlation (Xu et al., 1994). Unlike LEED and RHEED, surface electron diffraction may be assumed to be approximately kinematical, providing the sample is tilted off-zone, (Tanishiro and Takayanagi, 1989; Twesten and Gibson, 1994), and therefore much more straightforward to analyze. It should be noted that the diffraction patterns obtained are two dimensional, meaning that only atomic movement in the plane of the surface can be assessed directly from one diffraction pattern. This can lead to difficulties, for example, in deciding whether one atom sits above or below another.

As with the other methods discussed, this work must be carried out in UHV. As with other forms of microscopy, there is a possibility that the electron beam will cause radiation damage to the specimen, and, in this case, may also provide sufficient energy for surface rearrangements to occur.

ANALYTICAL METHODS

Current Techniques

The techniques outlined above provide the raw data for surface structure analysis. In the case of LEED, this

Fig. 4. Restored TED pattern from a Si-(111)-(7×7) surface. Experimental intensities from the Si-(111)-(7×7) surface were used, with the bulk spots interpolated from these. Bulk spots are indexed.
is done by means of “trial and error,” the accuracy of which depends on the quality of the initial model. Another approach is that of “direct methods” (see, for example, Giacovazzo, 1980; Schenk, 1991), in which mathematical relationships are used to derive probable phases from the observed structure factor amplitudes. Direct methods are applicable to both X-ray and electron diffraction data, and both will be considered here.

Diffraction data from surfaces are inherently problematic for direct methods (Marks and Landree, 1997), since there is usually a degree of correlation between the surface diffraction pattern, and some bulk diffraction spots. Note that the bulk spots will be many times more intense than the surface spots. Any degree of overlap between a surface and bulk diffraction spot means that the surface spot intensity cannot be measured accurately, thus leading to the possibility of either large errors in the data set, or, if many surface and bulk spots coincide, large numbers of missing reflections in the data. It should also be noted that surface diffraction patterns are inherently noisy. Finally, the exact atomic composition of a surface unit cell is rarely known, and while most direct methods are fairly tolerant of this, it should be borne in mind when analysis is carried out (Gilmore et al., 1997).

Of these problems, that of missing information is by far the most significant. In many cases, the strongest surface reflections coincide with the bulk reflections, and if these are removed, then even if all other reflections are assigned the correct phases, the holes in the data mean that the surface potential maps (for electrons-electron density maps for X-rays) will show negative areas. Even if the correct potential map is located, there is no guarantee that all of the atoms present in the structure will be found. Those that have not been displaced far from bulk positions may well be masked, leaving doubt as to their exact positions.

Despite these problems, it is possible to solve surface structures via direct methods calculations. An example of such a method that has been applied to surfaces with some success is the Maximum Entropy (ME) technique (Bricogne, 1984, 1988; Bricogne and Gilmore, 1990; Gilmore, 1996; Gilmore et al., 1990, 1993), discussed by Gilmore (1999). The method developed at Northwestern, a “Minimum Relative Entropy” (MRE) approach (Landree et al., 1997; Marks and Landree, 1997), has also proved very successful in analyzing surface structures (see Table 1), and the theoretical and practical aspects of this will be outlined below.
Theory of Minimum Relative Entropy

For a complete set of unitary structure factors in reciprocal space, $U(k)$, the Sayre equation (Sayre, 1952) states that, for non-overlapping identical atoms,

$$ U(k) = N \sum_{h} U(k - h) U(h). $$

If some initial phases are known, the Tangent Formula (Karle and Hauptman, 1956) may be used to generate new phases, with these new phases being put back into the original equation and their validity tested by some sort of figure of merit (FOM).

This process may be considered as a Picard iteration, and equation 1 can be better represented as

$$ U_{n+1}(k) = N \sum_{h} U_{n}(k - h) U(h), $$

where $U_{n}(k)$ contains the $n$'th iteration estimated phase. This may be rewritten in terms of real space, such that

$$ u_{n+1}(r) = \hat{O}[u_{n}(r)], $$

where $\hat{O}$ is some sharpening operator operating on a given real space potential map, $u_{n}(r)$. The ideal operator will enhance strong features in $u_{n}(r)$ while dampening weaker ones. We define the relative entropy, $S_{r}(r)$ (Cover and Thomas, 1991) as

$$ S_{r}(r) = u_{n}(r) \ln[u_{n}(r)/e^{<u_{n}(r)>}] $$
$$ + <u_{n}(r)> \text{ if } u_{n}(r) > 0, $$
$$ = <u_{n}(r)> \text{ if } u_{n}(r) < 0, $$

where $<u_{n}(r)>$ is the mean value of $u_{n}(r)$. The real space potential map, $u_{n}(r)$, contains maximum information, and has maximum entropy, when $u_{n}(r) = <u_{n}(r)>$, so as maximum information is approached, $S_{r}(r) \rightarrow 0$. Hence, the most probable solution will satisfy

$$ u_{n+1}(r) \approx u_{n}(r) \ln[u_{n}(r)/<u_{n}(r)>] + <u_{n}(r)> $$

and, thus, the relative entropy may be used as a self-consistent sharpening operator. This operator will minimize the background present in $u_{n}(r)$ while at the same time sharpening the areas that contain information.

A window function, $W(k)$, is chosen to satisfy

$$ w(r) = \text{FFT}[W(k)] = \alpha \hat{O}[w(r)], $$

where $\alpha$ is some constant. This gives any single non-overlapping atom a relative entropy value of zero.
Application of this window gives windowed unitary structure factors,

$$U'(k) = W(k)U(k).$$  \hspace{1cm} (7)$$

The $U'$ in equation 7 still satisfies equation 5, as long as a multiplicative constant for non-overlapping individual atoms is considered.

The last item required is a figure of merit (FOM), which will allow the solutions obtained to be ranked in some way. An effective FOM is given by

$$FOM = \frac{\sum' |U_n(k) - \beta U_{n-1}(k)|/|U_n(k)|}{\sum |U_n(k)|},$$ \hspace{1cm} (8)

where the sum $\sum'$ is taken over all reflections except $k = 0$, and $\beta$ is chosen to minimize the FOM.

The $U_{n-1}(k)$ are corrected at the end of each cycle so that they have the experimentally obtained moduli. The iteration ends when the FOM stops decreasing. Thus, the lower the FOM value, the better the solution.

Fig. 7. Contour map showing the top view of the best solution obtained by MRE phase extension method for Si-(111)-(7×7) surface using plane group p6mm, with (7×7) unit cell marked. All atom positions in the first three layers are resolved.
Unlike the ME method, the FOM is a measure of the consistency of the solution with the operator. In other words, if the operator changes phases and/or amplitudes dramatically, this will be reflected in a high FOM.

**Genetic Algorithm**

To generate the initial set of phases, and to obtain optimized solutions, a genetic algorithm is used (Chang and Lewis, 1994; Davis, 1987; Goldberg, 1989; Gutowski, 1994; Landree et al., 1997; Xiao and Williams, 1993). This starts with a set, \( S \), of structure factors. A

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**Fig. 8.** Top view of best solutions obtained for TiO\(_2\) in pm plane group with varying numbers of Ti atoms. Left-hand column shows the positions of the Ti atoms; the right-hand column shows the position of the unit cell relative to these atoms. White areas correspond to possible atom sites. \( a \): 5 Ti atoms in the unit cell. \( b \): 6 Ti atoms in the unit cell. \( c \): 4 Ti atoms in the unit cell.

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**TABLE 2.** \( \chi^2 \) Values for Varying Unit Cell Contents of TiO\(_2\) Surface

<table>
<thead>
<tr>
<th>Number and type of atom in unit cell</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 ( \times ) Ti</td>
<td>3.45</td>
</tr>
<tr>
<td>4 ( \times ) Ti; 2 ( \times ) O</td>
<td>2.27</td>
</tr>
<tr>
<td>4 ( \times ) Ti; 3 ( \times ) O</td>
<td>2.17</td>
</tr>
<tr>
<td>4 ( \times ) Ti; 4 ( \times ) O</td>
<td>2.78</td>
</tr>
<tr>
<td>4 ( \times ) Ti; 6 ( \times ) O</td>
<td>1.12</td>
</tr>
<tr>
<td>4 ( \times ) Ti; 7 ( \times ) O</td>
<td>0.937</td>
</tr>
<tr>
<td>4 ( \times ) Ti; 8 ( \times ) O</td>
<td>1.054</td>
</tr>
<tr>
<td>4 ( \times ) Ti; 9 ( \times ) O</td>
<td>0.805</td>
</tr>
</tbody>
</table>
subset of this set, $s$, containing the strongest structure factors that have been assigned phases (see Fig. 2), is calculated. These phase values are encoded into a set of bits, which is called a gene. Each gene represents the phase for a single reflection, and typically between 10 and 20% of the total number of reflections present will have their phases assigned. The genes are then stored as a single string of bits, called a chromosome.

Starting with a population, $N$, of chromosomes, the remaining phases are calculated using the MRE approach described above, and a FOM is obtained. Natural selection then takes place, in which parents are chosen for the next generation calculation. Parents with low FOM values are favored. Pairs of parents produce children by cross-linking. This is where a location along the parent chromosomes is chosen at random, and bits on one side of this location are interchanged. Each new population thus produced is then subject to a degree of mutation, where bit values are randomly changed from 0 to 1, or vice versa. New phases are calculated for this population, and the FOMs obtained are evaluated as before in order to produce children for the next generation (see Fig. 2).

This relatively simple process is in fact an extremely powerful tool that is capable of global searching for minima highly efficiently. This characteristic of a genetic algorithm is attributed to the way the search

Fig. 9. a: Original structural model for TiO$_2$ (Zshack et al., 1992). b: Model obtained from the MRE calculation. The Ti atoms lie at the center of the octahedra, with O atoms at each corner. The new model fits the experimental data well, and is in agreement with known TiO$_2$ bulk structures. The dark gray octahedra show how the new model differs from the previous one.
accounts for schemata. A schemata is a similarity template that describes a subset of chromosomes that have similarities at certain positions. For the simplest case, when the number of parents is equal to the number of children, the algorithm processes on the order of $N^3$ schemata, even though only $N$ new FOM values are calculated for each generation. If cross-linking breaks a schemata, the search will be slowed down, so short, favorable schemata are better preserved across generations. As a schemata represents a phase choice, finding an effective schemata is akin to finding good phase choices, and thus the genetic algorithm is an excellent search engine for this type of problem.

Once solutions are obtained from the genetic algorithm search, the structure is completed for the best of these using a heavy-atom holography (Marks and Plass, 1995) algorithm that utilizes the relationship between a known wave, $C(u)$, and an unknown wave, $w(u)$, in the diffraction plane to determine the phase, by minimizing

$$P^2 = |I(u) - \alpha|\psi(u) + w(u)|^2$$ \hspace{1cm} (9)

where $I(u)$ is the experimental diffraction intensities of a single domain and $\alpha$ is the scaling factor determined by $\chi^2$ minimization. The Fourier transform of $w(u)$ is then an approximation of the residual wave, and atomic locations may be found by inspection of the image.

Solutions are then discriminated based upon physical or chemical considerations, and final atomic positions refined using a $\chi^2$ minimization technique, where

$$\chi^2 = \frac{[1/(M - N)|F_{\text{exp}}|^2 - \alpha|F_{\text{calc}}|^2]^2}{\Sigma \sigma^2}$$ \hspace{1cm} (10)

where $M$ is the number of points, $N$ the number of variables, $F_{\text{exp}}$ represents the experimentally observed amplitudes, $\alpha$ is a variable, $F_{\text{calc}}$ the calculated amplitudes, and $\sigma$ represents the errors involved in the experiment. A $\chi^2$ value of 1.00 represents perfect agreement between the observed and calculated structures. If a Poisson error distribution is assumed, this may be expressed as the crystallographic R-factor,

$$R_n = \frac{[1/(M - N)|F_{\text{exp}} - \alpha|F_{\text{calc}}|^n\Sigma|F_{\text{exp}}|^n]}{\Sigma \sigma^2}$$ \hspace{1cm} (11)

which may also be used as a criterion for determining the “correctness” of a structure.

**EXAMPLES**

Thus far, the results of this method of analysis are promising. It has been able to solve both known and unknown structures for both X-ray and electron data sets. Before discussing the examples, a brief explanation of surface notation is necessary (see also Fig. 3). The most common method of surface notation allows the surface structure to be defined relative to that of the two-dimensional surface unit cell. If the surface contains an atomic arrangement identical to that of a bulk termination, it is designated $(1 \times 1)$. If, however, the surface may be represented by an arrangement of $m$...
unit cell lengths and n unit cell widths, it is assigned \((m \times n)\). In addition, the face of the surface concerned is listed, and it is usual to list the adsorbate, if one exists, so that, for example, \(\text{Si (111)-(2 \times 1)}\) Au, refers to gold on the (111) face of silicon, two surface unit cells in length and one in width.

\(\text{Si (111)-(7 \times 7)}\)

The Si (111)-(7 \times 7) structure has been one of the great mysteries of surface science. It proved completely unsolvable for many years, until a model was put forward in 1985 (Takayanagi et al., 1985a,b) as a result of a Patterson analysis of electron diffraction data. This surface was analyzed via the ME technique, and also by the MRE method outlined above (Gilmore et al., 1997). One hundred and ninety-three unique diffraction reflections (Fig. 4) with resolution out to 0.6Å were used in the analysis, and 41 phases were obtained from a high-resolution image. Despite the relatively large data set, the calculation was not straightforward due to the very strong missing reflections. The best structural-analysis results were obtained when a phase-extension calculation was carried out, using the 41 experimentally determined phases.

The top view of the correct structure is shown in Fig. 5. It should be noted that the fourth layer of atoms is complete, but half of the atoms in this layer lie directly under atoms in the layers above, and so cannot be seen in this projection of the surface. The best solution found by means of a ME phase-extension calculation is shown in Figure 6. Notice that not all of the atoms have been found. Figure 7 shows the best solution obtained from a MRE analysis, in which all atoms in the top three layers have been resolved. Both solutions agree with the model.

\(\text{TiO}_2 (100)-(1 \times 3)\)

Titanium dioxide exhibits useful catalytic properties, which have led to renewed interest in the elucidation of the surface structure of the compound (Chung et al., 1977; Munnix and Schmeits, 1984; Oliver et al., 1994). A model was proposed in 1992 (Zschack et al., 1992), based on a Patterson analysis of grazing-incidence X-ray-diffraction data. This model, though chemically feasible, agreed poorly with the experimental data.

A better solution was found using direct methods (Landree et al., 1997). The same data set of 19 unique reflections as that from which the original model was proposed was used, meaning that much information was missing, which led to problems in the refinement stage of the calculation. Four plane groups were possible, and the calculation was run for each of them. After obtaining solutions for each possible plane group, a refinement and heavy-atom holography calculation was carried out, with titanium atoms being placed at sites in the unit cells obtained. A crystallographic
R-factor calculation was carried out for each of these solutions, and only those with an R-factor of less than 0.41 were kept. All of the solutions that met this criterion fell into the same plane group, \( pm \) (Fig. 8).

Further heavy-atom holography studies showed that the best agreement between experimental and theoretical data was obtained when 4 titanium atoms and 6–8 oxygen atoms were present in the \((1 \times 3)\) unit cell (see Table 2), though with so few reflections in the data set, it is impossible to carry out a rigorous analysis for oxygen positions. Figure 9 shows a schematic of the original model and the model found by direct methods. The model found via direct methods calculations shows non-stoichiometric features that have been documented for bulk TiO$_2$, and it is reasonable to assume that this surface reconstruction reflects an ordering of these features at the surface. In addition, the new model also fits STM data.

**Si (111)-(6 \times 6)-Au**

This surface has been studied, but the full structure was not understood (Dornisch et al., 1991). The MRE method was used with X-ray diffraction data (Marks et al., 1998a), and again the diffraction pattern suggested...
more than one possible plane group. Figure 10 shows one of the top maps obtained from the MRE direct method calculation, with plane group \( p3 \) used. As in the case of TiO\(_2\), heavy-atom holography was used in conjunction with an iterative refinement process to determine the full structure, which is shown in Figure 11.

**Si (111)-(4 \times 1)-In**

This is another example of a structure that had been extensively studied but not solved (Nogami et al., 1987; Stevens et al., 1993). In particular, AES and photoemission studies suggested that four indium atoms were present in the \((4 \times 1)\) unit cell (Abukawa et al., 1995; Nakamura et al., 1991), while ion-scattering spectrometry appeared to suggest that only two indium ions were present (Cornelison et al., 1991).

Electron-diffraction data gave 27 unique reflections (Fig. 12), with four possible plane groups. The MRE calculation was run for each possibility (Collazo-Davila et al., 1997), and from the top solutions the plane groups \( pm \) and \( p2mg \) were found to be the most likely (Fig. 13). After refinement and heavy-atom holography, the final structure obtained exhibited \( pm \) symmetry, with two indium atoms in the unit cell (Fig. 14).

These examples are representative of the different types of problems that surface data present for direct methods.

**DISCUSSION**

Direct methods, as applied to surface structure analysis, have emerged in the last year as an exciting new tool in this important area of research. The technique still has to be fully optimized, and is perhaps less straightforward than when used for bulk structure analysis, but early difficulties, such as incomplete data, are becoming less problematic with the addition of a technique that interpolates missing reflections in a data set. Expansion of this method into three-dimensional problems has begun recently, though in the case of electron diffraction, this is limited by the difficulty in obtaining data in three dimensions. A three-dimensional approach could prove most useful in the consideration of subsurface rearrangements, since only diffraction data will give information about this. It should also be noted that, thus far, the electron data used have
given maps of poorer quality that those produced from X-ray data, suggesting that a wholly kinematical approach to electron scattering is not completely correct.

In conclusion, direct methods for surface analysis is a specialized technique at the time of writing, and it may take some time for its advantages to be fully understood. Methods such as LEED and PEH are also extremely useful in surface structure analysis, while AES, STM, SIMS, XPS, RHEED, and RBS also prove to be of great use in providing starting points for models of surfaces. While some work is required to ensure that the data used are of good quality, there is no reason to doubt that direct methods of structure analysis will become as common a tool to the surface scientist in years to come as it is to crystallographers today.

REFERENCES


Chung YW, Lo WJ, Somorjai GA. 1977. Low energy electron diffraction and electron spectroscopy studies of the clean (110) and (100) titanium dioxide (Rutile) crystal surfaces. Surf Sci 64:588–602.


